MHF 4U1 Test: Trigonometry

Part A) Knowledge and Understanding

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. The value of $\cos(5\pi/12)$ is:

   a. $\sin(5\pi/13)$
   b. $\cos(7\pi/12)$
   c. $\cos(\pi/2 - 5\pi/12)$
   d. $\sin(\pi/2 - 5\pi/12)$

   d. Use $\cos x = \sin(\pi/2 - x)$.

2. Which of the following compound angles for $11\pi/12$ will not allow the use of special triangles?

   a. $15\pi/12 - 4\pi/12$
   b. $7\pi/12 + 4\pi/12$
   c. $8\pi/12 + 3\pi/12$
   d. $20\pi/12 - 9\pi/12$

   b. $7\pi/12 = 105°$, whose reference angle is 75°, not 30°, 45°, or 60°.

3. Determine the exact degree measure for $3\pi/5$.

   a. 108°
   b. 226°
   c. 36°
   d. 145°

   a. $(3\pi/5)(180°/\pi) = 108°$.

4. Determine the approximate degree measure for an angle of 1.32 radians.

   a. 136.4°
   b. 4.2°
   c. 75.6°
   d. 2.4°

   c. $1.32(180°/\pi) \approx 75.6°$.

5. Determine the exact value of $\cos(\pi/4)$.

   a. $1/\sqrt{2}$
   b. $\sqrt{2}$
   c. $\sqrt{3}/2$
   d. $1/2$

   a. $\cos(\pi/4) = \sqrt{2}/2 = 1/\sqrt{2}$.

6. The point $(-4, 3)$ is on the terminal arm of angle θ in standard position. The value of $\cos\theta$ is:

   a. $3/5$
   b. $-3/5$
   c. $-4/3$
   d. $-4/5$

   d. $r = \sqrt{(16 + 9)} = 5$, so $\cos\theta = x/r = -4/5$.

7. Determine the exact value of sec π.

a. 0
c. 1
b. −1
d. undefined

b. sec π = 1/cos π = 1/(−1) = −1.

8. Use your calculator to determine the value of cos(1.54), to three decimal places.

a. 0.031
c. 0.034
b. 0.054
d. 0.005

a. In radian mode, cos(1.54) ≈ 0.030791, which rounds to 0.031.

9. Use your calculator to determine the value of cot(4.25), to three decimal places.

a. 2.006
c. 0.074
b. 0.498
d. 13.457

b. In radian mode, 1/tan(4.25) ≈ 0.498428, which rounds to 0.498.

10. 1/8 of a revolution represents how many radians?

a. π/8
c. π/4
b. 135
d. 67.5

c. (1/8)(2π) = π/4.

Short Answer

1. Angle θ lies in the third quadrant such that $\cos\theta = -3/5$. Determine an approximate measure for θ in radians, to the nearest hundredth.

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The reference angle is $\alpha = \cos^{-1}(3/5) \approx 0.92730$ radians.
In Quadrant III, $\theta = \pi + \alpha$.
$\theta \approx 4.07$ radians.

2. Determine an exact value for each of the following:

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(a) $[\cot^2(\pi/4)\csc^2(\pi/3)] / [\cos^2(\pi/3) - \sec^2(2\pi)]$

(b) $\sin(5\pi/4) - \cos(11\pi/6)\cot(\pi/3)$

(a) $\cot^2(\pi/4) = 1$, $\csc^2(\pi/3) = (2/\sqrt{3})^2 = 4/3$.
$\cos^2(\pi/3) = 1/4$ and $\sec^2(2\pi) = 1$.
The quotient is $(4/3)/(-3/4) =$ **$-16/9$.**

(b) $\sin(5\pi/4) = -\sqrt{2}/2$, $\cos(11\pi/6) = \sqrt{3}/2$, and $\cot(\pi/3) = 1/\sqrt{3}$.
Substitute: $-\sqrt{2}/2 - (\sqrt{3}/2)(1/\sqrt{3}) =$ **$(-\sqrt{2} - 1)/2$.**

Part B) Applications

1. Using a co-related angle formula, find the exact value of: /3

$\tan(5\pi/3)$

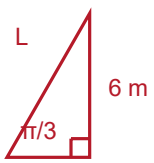
$5\pi/3 = 3\pi/2 + \pi/6$. Using $\tan(3\pi/2 + x) = -\cot x$,
$\tan(5\pi/3) = -\cot(\pi/6) = -\sqrt{3}$.

2. The arc length of the sector of a grandfather clock is 50 cm, and the angle of the sector is 120°. Determine the radius of the clock, expressing your answer in terms of π. /3

Convert 120° to $2\pi/3$ radians. Using $s = r\theta$,
$r = 50/(2\pi/3) = 75/\pi$ cm.

3. A skateboard ramp is built with an incline angle of $\pi/3$. If the height of the ramp is 6 m, determine the exact length of the ramp. Include a diagram. /3

Let L be the ramp length. The vertical height is opposite the angle.
$\sin(\pi/3) = 6/L$, so **$L = 6/(\sqrt{3}/2) = 4\sqrt{3}$ m.**



4. Evaluate $\sin(2x)$ if $\cos x = -4/5$ and $\pi/2 \leq x \leq \pi$. /3

x is in Quadrant II, so sin x is positive.
$\sin x = \sqrt{(1 - (-4/5)^2)} = 3/5$.
$\sin(2x) = 2 \sin x \cos x = 2(3/5)(-4/5) = -24/25$.

5. A merry-go-round rotates 180 times per hour. Determine the exact angular velocity in radians per second and in degrees per second. /3

180 revolutions/hour = 180/3600 = 1/20 revolution/second.
$\omega = (1/20)(2\pi) = \pi/10$ rad/s.
$\omega = (1/20)(360°) = 18°$/s.

6. Prove the following trigonometric identity: $(\csc x - \cot x)^2 = (1 - \cos x)/(1 + \cos x)$ /4

On the common domain $\sin x \neq 0$,
LHS $= [1/\sin x - \cos x/\sin x]^2$
$= (1 - \cos x)^2 / \sin^2 x$
$= (1 - \cos x)^2 / [(1 - \cos x)(1 + \cos x)]$
$= (1 - \cos x)/(1 + \cos x) =$ RHS.

Part C) Communication

1. If you are given an angle θ, that lies somewhere in the interval $\pi/2 \leq \theta \leq 2\pi$, how would you determine the values of the primary trigonometric ratios for this angle?

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Find the reference angle α, evaluate the ratios at α, then apply quadrant signs:

Quadrant II: $\alpha = \pi - \theta$; sin positive, cos and tan negative.
Quadrant III: $\alpha = \theta - \pi$; sin and cos negative, tan positive.
Quadrant IV: $\alpha = 2\pi - \theta$; cos positive, sin and tan negative.

For angles on the axes, use the unit-circle point $(\cos \theta, \sin \theta)$ directly.
Compute $\tan \theta = \sin \theta/\cos \theta$ when $\cos \theta \neq 0$. It is undefined otherwise.

Part D) Thinking, Inquiry and Problem Solving

1. Determine a formula for cos 3θ, that is only in terms of cos θ.

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$\cos(3\theta) = \cos(2\theta + \theta)$
$= \cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta$
$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$
$\mathbf{= 4\cos^3\theta - 3\cos\theta.}$