MHF4U1 Assignment: Trigonometry

Part A) Knowledge and Understanding

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. The exact radian measure for an angle of 135° is:
   a. $\pi/4$
   b. $3\pi/4$
   c. $\pi/2$
   d. π

   b. $135° \times \pi/180° = 3\pi/4$.

2. 7/8 of a revolution represents how many radians?
   a. $7\pi/4$
   b. 315
   c. $7\pi/8$
   d. 157.5

   a. $(7/8)(2\pi) = 7\pi/4$.

3. Determine the exact degree measure for $2\pi/5$.
   a. 36°
   b. 226°
   c. 72°
   d. 144°

   c. $(2\pi/5)(180°/\pi) = 72°$.

4. Determine the approximate degree measure for an angle of 2.54 radians.
   a. 0.044°
   b. 457.2°
   c. 0.0254°
   d. 145.5°

   d. $2.54(180°/\pi) \approx 145.5°$.

5. Determine the exact value of $\cos(\pi/4)$.
   a. $1/\sqrt{2}$
   b. $\sqrt{2}$
   c. $\sqrt{3}/2$
   d. 1/2

   a. $\cos(\pi/4) = \sqrt{2}/2 = 1/\sqrt{2}$.

6. Determine the exact value of $\sin(5\pi/6)$.
   a. $2/\sqrt{3}$
   b. 2
   c. $\sqrt{3}/2$
   d. 1/2

   d. $\sin(5\pi/6) = \sin(\pi/6) = 1/2$.

7. Determine the exact value of $\cot(5\pi/3)$.
   a. $1/\sqrt{3}$
   b. $\sqrt{3}$
   c. $-1/\sqrt{3}$
   d. 1/2

   c. $\cos(5\pi/3)/\sin(5\pi/3) = (1/2)/(-\sqrt{3}/2) = -1/\sqrt{3}$.

8. Determine the exact value of $\sin \pi$.

a. 0
c. 1
b. −1
d. undefined

a. The unit-circle point at π is (−1, 0), so sin π = 0.

9. Determine the exact value of $\sec(3\pi/2)$.

a. 0
c. 1
b. −1
d. undefined

d. cos(3π/2) = 0, so its reciprocal is undefined.

10. Use your calculator to determine the value of sin 3.11, to three decimal places.

a. 0.031
c. 0.054
b. 0.032
d. 0.005

b. In radian mode, sin(3.11) ≈ 0.031587, which rounds to 0.032.

11. Use your calculator to determine the value of csc 4.37, to three decimal places.

a. 0.076
c. −1.062
b. 13.124
d. −0.942

c. In radian mode, 1/sin(4.37) ≈ −1.062.

Part B) Communication

A student claims that the identity $\sin^2 x + \cos^2 x = 1$ can be used to evaluate $\sin^2(2\pi/3)$ without a calculator. Explain clearly and mathematically how this identity can be used to determine $\sin^2(2\pi/3)$. Justify each step.

/3

Substitute $x = 2\pi/3$ and rearrange:
$\sin^2(2\pi/3) = 1 - \cos^2(2\pi/3)$.
The angle $2\pi/3$ is in Quadrant II, with reference angle $\pi/3$.
Therefore $\cos(2\pi/3) = -\cos(\pi/3) = -1/2$.
$\sin^2(2\pi/3) = 1 - (-1/2)^2 = 3/4$.

Part C) Applications

1. Find the exact value of $\sin(7\pi/12)$ using a compound angle formula. /4

$7\pi/12 = \pi/4 + \pi/3.$
$\sin(7\pi/12) = \sin(\pi/4)\cos(\pi/3) + \cos(\pi/4)\sin(\pi/3)$
$= (\sqrt{2}/2)(1/2) + (\sqrt{2}/2)(\sqrt{3}/2)$
$= (\sqrt{6} + \sqrt{2})/4.$

2. Find the exact value of $\cos(4\pi/3)$ using a co-related angle formula. /3

$4\pi/3 = 3\pi/2 - \pi/6.$
Using $\cos(3\pi/2 - x) = -\sin x,$
$\cos(4\pi/3) = -\sin(\pi/6) = -1/2.$

3. Determine the exact value of the following: /3
$(\cot(\pi/3) + \sec(\pi/6))(\sin(\pi/2) - (2/3)\sin(\pi/6))$.

Substitute the special-angle values:
$= (1/\sqrt{3} + 2/\sqrt{3})(1 - (2/3)(1/2))$
$= \sqrt{3}(2/3)$
$= 2\sqrt{3}/3$.

4. An athletic facility has a circular outdoor running track with a diameter of 32 m. A runner ran along the arc of the track for 60 m. What is the sector angle, in degrees, from the start to the finish of her run? /3

The radius is 32/2 = 16 m.
Using $s = r\theta$, $\theta = 60/16 = 15/4$ radians.
$\theta = (15/4)(180°/\pi) = 675°/\pi \approx 214.9°$.
The runner covers more than half a lap, so the angle exceeds 180°.

5. The "Big Wheel" on The Price is Right rotates 12 times per minute. Determine its angular velocity in radians per second, in exact form and to the nearest hundredth. /3

$\omega = (12 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})$.
$\omega = 2\pi/5$ rad/s ≈ 1.26 rad/s.

6. If $\sin A = 3/5$ for $\pi/2 \le A \le \pi$, and $\tan B = -3/4$ for $3\pi/2 \le B \le 2\pi$, then find the values of: /7

(a) $\cos(A + B)$

(b) $\sin 3B$

A is in Quadrant II, so $\cos A = -4/5$.
B is in Quadrant IV, so $\sin B = -3/5$ and $\cos B = 4/5$.

(a) $\cos(A + B) = \cos A \cos B - \sin A \sin B$
$= (-4/5)(4/5) - (3/5)(-3/5)$
$= -7/25$.

(b) $\sin(2B) = 2(-3/5)(4/5) = -24/25$.
$\cos(2B) = (4/5)^2 - (-3/5)^2 = 7/25$.
$\sin(3B) = \sin(2B)\cos B + \cos(2B)\sin B$
$= (-24/25)(4/5) + (7/25)(-3/5)$
$= -117/125$.

7. Express as a single trigonometric function, and then determine the exact value. /3

$\sin(11\pi/6)\cos(2\pi/3) - \cos(11\pi/6)\sin(2\pi/3)$

Using $\sin(u - v) = \sin u \cos v - \cos u \sin v$:
$= \sin(11\pi/6 - 2\pi/3) = \sin(7\pi/6)$
$= -1/2$.

Part D) Thinking, Inquiry and Problem Solving

Prove the following trigonometric identities:

1. $$\frac{\sin(\pi - x)}{\tan(\pi + x)} \times \frac{\cot(\pi/2 - x)}{\tan(\pi/2 + x)} \times \frac{\cos(2\pi - x)}{\sin(-x)} = \sin x$$ /4

On the common domain $\sin x \neq 0$ and $\cos x \neq 0$, use the related-angle identities:

LHS = $(\sin x/\tan x)(\tan x/(-\cot x))(\cos x/(-\sin x))$
= $(\sin x/\tan x)(-\tan^2 x)(-\cot x)$
= $\sin x \cdot \tan x \cdot \cot x$
= sin x = RHS.

2. $\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$ /3

For $\cos x \neq 0$,
LHS = $(\sin^2 x + \cos^2 x) + \tan^2 x$
= $1 + \sin^2 x/\cos^2 x$
= $(\cos^2 x + \sin^2 x)/\cos^2 x$
= $1/\cos^2 x = \sec^2 x$ = RHS.