

Part A - Multiple Choice

Question	Answer	Reason
1	C	$f(-2) = 16 + 8 + 6 + 7 = 37$.
2	D	All three division statements/restrictions are valid.
3	C	$f(-\frac{1}{2}) = 0$, so $2x + 1$ is a factor.
4	A	$f(1/3) = 1/9 + 8/9 + k/3 - 2 = 0$, so $k = 3$.
5	B	$x^3 - 6x^2 - 6x - 7 = (x - 7)(x^2 + x + 1)$.
6	C	Treat x^2 as a variable: $(x^2 + 1)(x^2 - 3)$.
7	B	$2x^4 + x^3 + 4x + 2 = (2x + 1)(x^3 + 2)$; $x = -\frac{1}{2}$.
8	D	Each equation has a nonzero leading coefficient and the required zeros/multiplicities.

Part A - True/False

Question	Answer	Reason
9	True	The remainder has degree less than the divisor (a nonzero remainder is constant for a linear divisor).
10	True	This is the factor theorem.
11	False	A polynomial that does not factor over the rationals can still have real roots; e.g., $x^2 - 2 = 0$.
12	False	Polynomial inequalities generally give intervals of solutions, not three individual values.

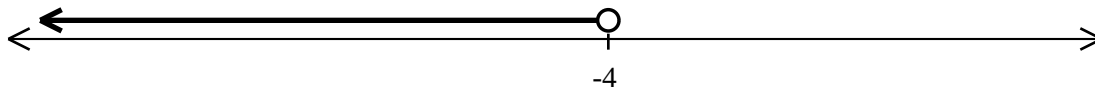
Part A - Short Answer

$$3x(2x - 1) - 8 > 2x + 6(x^2 + 2)$$

$$6x^2 - 3x - 8 > 6x^2 + 2x + 12$$

$$-5x > 20$$

$x < -4$. Reverse the inequality when dividing by -5.



Part B - Applications

1. Division: First write the dividend in descending powers: $2x^3 + 3x^2 - x + 5$.

Term in quotient	Multiply and subtract	Remaining expression
x^2	$x^2(2x - 3) = 2x^3 - 3x^2$	$6x^2 - x + 5$
$3x$	$3x(2x - 3) = 6x^2 - 9x$	$8x + 5$
4	$4(2x - 3) = 8x - 12$	17

Quotient: $x^2 + 3x + 4$. **Remainder:** 17.

$$(2x^3 + 3x^2 - x + 5)/(2x - 3) = x^2 + 3x + 4 + 17/(2x - 3), x \neq 3/2.$$

2. Solve: $3x^4 + 5x^3 - 13x^2 - x + 6 = 0$

$P(1) = 0$, so divide by $x - 1$:

$$(x - 1)(3x^3 + 8x^2 - 5x - 6) = 0.$$

The cubic is zero at -3, so divide by $x + 3$:

$$(x - 1)(x + 3)(3x^2 - x - 2) = 0$$

$$(x - 1)^2(x + 3)(3x + 2) = 0$$

$x = 1$ (order 2), -3, -2/3.

Part B - Applications (continued)

3a. Difference of cubes:

$$125x^9 - (1/27)y^{15} = (5x^3)^3 - ((1/3)y^5)^3$$

$$\text{Use } a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

$$(5x^3 - (1/3)y^5)(25x^6 + (5/3)x^3y^5 + (1/9)y^{10}).$$

3b. Factoring by grouping:

$$6x^3 - 5x^2 - 6x + 5$$

$$= x^2(6x - 5) - (6x - 5)$$

$$= (x^2 - 1)(6x - 5)$$

$$= (x - 1)(x + 1)(6x - 5).$$

4. Equal remainders:

For $2y - 1$, substitute $y = 1/2$:

$$R_2 = 6(1/2)^3 - 3(1/2)^2 + 2(1/2) + 7 = 8.$$

For $x + 3$, substitute $x = -3$:

$$R_1 = 2(-3)^3 + 4(-3)^2 - k(-3) + 5 = 3k - 13.$$

$$\text{Set them equal: } 3k - 13 = 8 \Rightarrow 3k = 21 \Rightarrow \mathbf{k = 7}.$$

5. Polynomial inequality:

$$9x^3 - 16x \leq 0$$

$$x(3x - 4)(3x + 4) \leq 0.$$

Critical values: $-4/3, 0, 4/3$.

Interval	$(-\infty, -4/3)$	$(-4/3, 0)$	$(0, 4/3)$	$(4/3, \infty)$
Product sign	-	+	-	+

Select negative intervals and include each zero.

$$\mathbf{x \in (-\infty, -4/3] \cup [0, 4/3]}.$$

Part C - Communication

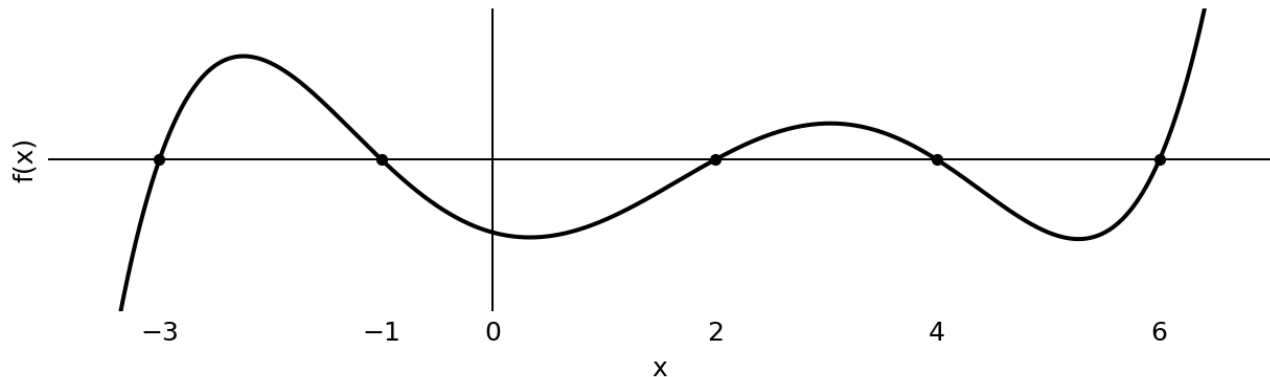
1. $x = 0, 3/4, -1$.

2. less than.

3. **Dividend/Divisor = Quotient + Remainder/Divisor**, with the divisor $\neq 0$.

4. One possible function is $f(x) = (x + 3)(x + 1)(x - 2)(x - 4)(x - 6)$.

It crosses at -3, -1, 2, 4, and 6, and is positive on the requested intervals. A positive vertical scaling gives the same sign pattern. The example sketch below uses a factor of 0.01 for display.



Part D - Thinking, Inquiry and Problem Solving

$$f(x) = ax^4 - 3x^3 - 63x^2 + 152x - b.$$

Use the zero $x = 5$:

$$0 = 625a - 375 - 1575 + 760 - b$$

$$0 = 625a - 1190 - b$$

$$\mathbf{b = 625a - 1190.}$$

Use the point $(-2, -560)$:

$$-560 = 16a + 24 - 252 - 304 - b$$

$$-560 = 16a - 532 - b$$

Substitute $b = 625a - 1190$:

$$-560 = -609a + 658$$

$$-1218 = -609a, \text{ so } \mathbf{a = 2.}$$

$$\mathbf{b = 625(2) - 1190 = 60.}$$

Final answer: $a = 2$ and $b = 60$.