

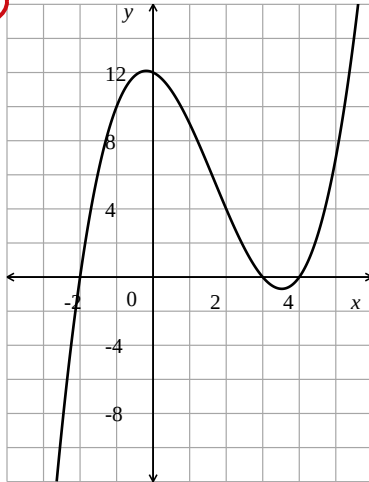
Part A) Knowledge and Understanding**Multiple Choice**

Identify the choice that best completes the statement or answers the question.

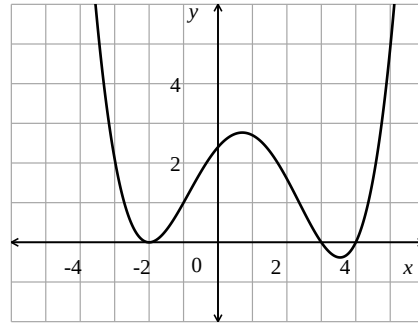
1. If $x^3 - 4x^2 + 5x - 6$ is divided by $x - 1$, then the restriction on x is
 - a. $x \neq -4$
 - b. $x \neq -1$
 - c. $x \neq 1$
 - d. no restrictions
2. What is the remainder when $x^4 + 2x^2 - 3x + 7$ is divided by $x + 2$?
 - a. 25
 - b. 13
 - c. 9
 - d. 37
3. Which of the following binomials is a factor of $8x^3 - 4x^2 - 2x + 1$?
 - a. $x - 1$
 - b. $x + 2$
 - c. $2x + 1$
 - d. $5x + 1$
4. Which set of values for x should be tested to determine the possible zeros of $x^3 - 2x^2 + 3x - 12$?
 - a. 1, 2, 3, 4, 6, and 12
 - b. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$, and ± 12
 - c. $\pm 1, \pm 2, \pm 3, \pm 4$, and ± 6
 - d. $\pm 2, \pm 3, \pm 4, \pm 6$, and ± 12
5. What is the remainder when $x^2 + 3x$ is divided by x ?
 - a. 0
 - b. -3
 - c. 4
 - d. you cannot divide by x
6. Which of the following is the fully factored form of $x^3 + 3x^2 - x - 3$?
 - a. $(x + 3)(x^2 - 1)$
 - b. $(x - 1)(x + 1)(x + 3)$
 - c. $x^2(x + 3) - (x + 3)$
 - d. $(x^2 - 1)(x - 3)$
7. A family of polynomials has equation $y = k(x - 4)(x + 1)^2$. What is an equation for the family member whose graph passes through the point (2, -54)?
 - a. $y = 3(x - 4)(x + 1)^2$
 - b. $y = -3(x - 4)(x + 1)^2$
 - c. $y = (1/70\,225)(x - 4)(x + 1)^2$
 - d. none of the above

8. Which of the following graphs of polynomial functions corresponds to a cubic polynomial equation with roots -2, 3, and 4?

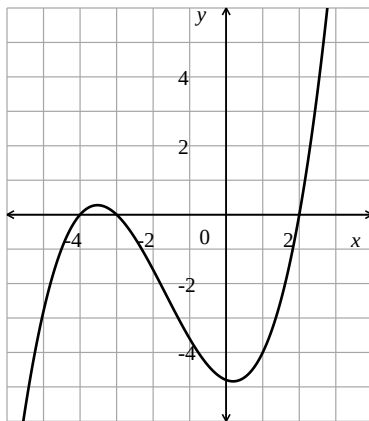
a.



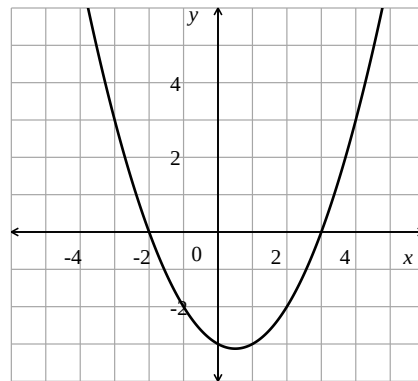
c.



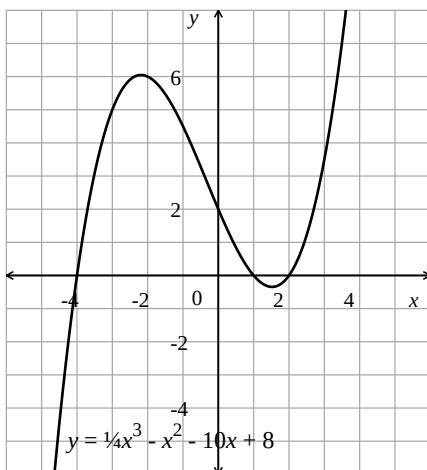
b.



d.



9. A graph of $y = \frac{1}{4}x^3 + x^2 - 10x + 8$ is shown. Use the graph to solve $x^3 + 4x^2 - 40x + 32 < 0$.



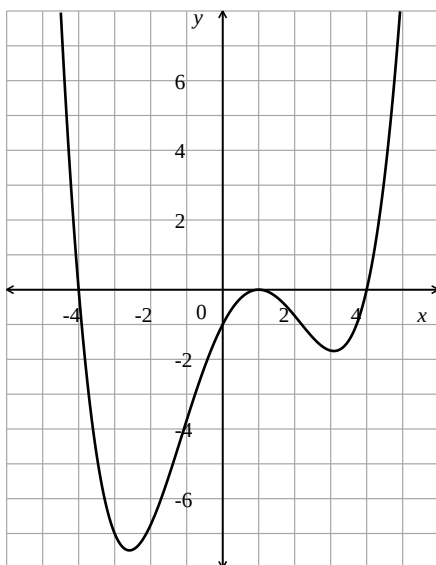
a. $x < -4$

b. $x < -4, 1 < x < 2$

c. $-4 < x < 1, x > 3$

d. $x < -4, 1 < x > 2$

10. What is an equation for the quartic function represented by this graph?



a. $y = (1/16)(x - 4)(x + 1)^2(x + 4)$

c. $y = 16(x - 4)(x + 1)^2(x + 4)$

b. $y = (1/16)(x - 4)(x - 1)^2(x + 4)$

d. $y = 16(x - 4)(x - 1)^2(x + 4)$

Short Answer

1. Use long division to divide:

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$$5x^4 - 7x^3 - 12x^2 - 1 \text{ by } x^2 - x + 2$$

$$5x^2(x^2 - x + 2) = 5x^4 - 5x^3 + 10x^2$$

$$\text{Subtract: } -2x^3 - 22x^2 + 0x - 1$$

$$-2x(x^2 - x + 2) = -2x^3 + 2x^2 - 4x$$

$$\text{Subtract: } -24x^2 + 4x - 1$$

$$-24(x^2 - x + 2) = -24x^2 + 24x - 48$$

$$\text{Remainder: } -20x + 47$$

$$5x^4 - 7x^3 - 12x^2 - 1 = (x^2 - x + 2)(5x^2 - 2x - 24) - 20x + 47$$

2. Divide $3x^3 - 2x^2 - 8$ by $x + 2$ using synthetic division and then write a division statement.

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$$\begin{array}{r|rrrr} -2 & 3 & -2 & 0 & -8 \\ & & -6 & 16 & -32 \\ \hline & 3 & -8 & 16 & -40 \end{array}$$

$$\text{Quotient: } 3x^2 - 8x + 16; \text{ remainder: } -40$$

$$3x^3 - 2x^2 - 8 = (x + 2)(3x^2 - 8x + 16) - 40$$

Part B) Applications

1. Using all known factoring skills, fully factor the following.

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a) $27x^3 + \frac{1}{8}y^6$

$$(3x)^3 + (\frac{1}{2}y^2)^3$$

$$= (3x + \frac{1}{2}y^2)(9x^2 - (3/2)xy^2 + \frac{1}{4}y^4)$$

b) $2x^4 + 7x^3 - 10x^2 - 32x$

$$= x(2x^3 + 7x^2 - 10x - 32)$$

$$= x(x + 2)(2x^2 + 3x - 16)$$

Irreducible over the rationals.

2. Solve the following polynomial equations. Exact answers only.

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a) $2x^3 + x^2 - 10x - 5 = 0$

$$x^2(2x + 1) - 5(2x + 1) = 0$$

$$(2x + 1)(x^2 - 5) = 0$$

$$x = -\frac{1}{2}, -\sqrt{5}, \sqrt{5}$$

b) $6x^3 + x^2 - 31x - 10 = 0$

$$6x^3 + x^2 - 31x + 10 = 0$$

$$(x - 2)(6x^2 + 13x - 5) = 0$$

$$(x - 2)(2x + 5)(3x - 1) = 0$$

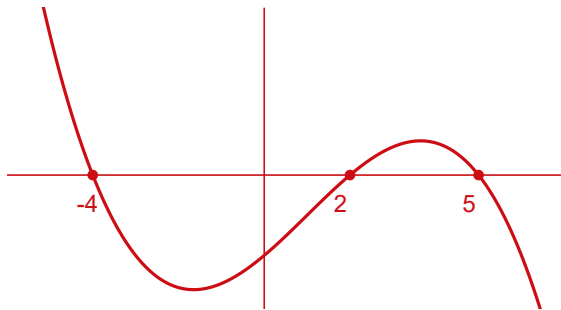
$$x = 2, -5/2, 1/3$$

3. Solve the following polynomial inequalities using each method once, making sure to show all work.

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a) $-3(x - 2)(x + 4)(x - 5) \leq 0$

Graphing method



Zeros: $-4, 2, 5$; cross at each zero.

On/below the axis: $-4 \leq x \leq 2$ or $x \geq 5$.

x in $[-4, 2]$ or $[5, \infty)$

b) $x^3 \geq 9x$

Interval method

$$x^3 - 9x \geq 0$$

$$x(x - 3)(x + 3) \geq 0$$

Intervals: $(-\infty, -3)$ $(-3, 0)$ $(0, 3)$ $(3, \infty)$

Signs: $-$ $+$ $-$ $+$

Include zeros since the inequality is $\geq$.

x in $[-3, 0]$ or $[3, \infty)$

4. For a family of cubic functions that have zeros of -2 and $4 \pm \sqrt{3}$

a) State the simplified general equation for any member of this family.

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$$f(x) = a(x + 2)[x - (4 + \sqrt{3})][x - (4 - \sqrt{3})]$$

$$= a(x + 2)[(x - 4)^2 - 3]$$

$$= a(x + 2)(x^2 - 8x + 13)$$

$$= a(x^3 - 6x^2 - 3x + 26), a \neq 0$$

b) State the specific equation for the member of this family that has a point of $(1, 24)$ on the graph.

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$$24 = a(1 - 6 - 3 + 26) = 18a$$

$$a = 4/3$$

$$f(x) = (4/3)(x^3 - 6x^2 - 3x + 26)$$

Part C) Communication

For # 1 - 3 complete each statement.

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1. If $P(-b/a) = 0$ for a polynomial $P(x)$, then $ax + b$ is a **factor** of $P(x)$.
2. Polynomial functions with the same zeros are said to belong to the same **family**.
3. If solving $P(x) \leq 0$ by using intervals and $P(x)$ has n number of zeros, then the number of intervals to be tested is **$n + 1$** .

Part D) Thinking, Inquiry and Problem Solving

Find any solutions that satisfy **both**: $x^3 + x^2 - 20x \leq 0$ and $x^4 - 9x^2 < 0$

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$$x^3 + x^2 - 20x = x(x + 5)(x - 4) \leq 0$$

First solution: $(-\infty, -5]$ or $[0, 4]$

$$x^4 - 9x^2 = x^2(x - 3)(x + 3) < 0$$

Second solution: $(-3, 0)$ or $(0, 3)$

Both must hold: take the intersection.

Solution: $0 < x < 3$

The endpoints 0 and 3 are excluded by the strict inequality.